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THERMAL DEFORMATION OF CONCENTRATORS IN AN AXISYMMETRIC TEMPERATURE FIELD

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16. Abstract

Axisymmetric thermal deformations of paraboloid mirrors, due to heating, are examined for a mirror with an optical axis oriented toward the sun. A governing differential equation is derived using Mushtari-Donnel-Vlasov simplifications, and a solution is presented which makes it possible to determine the principal deformation characteristics.

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THERMAL DEFORMATION OF CONCENTRATORS IN AN AXISYMMETRIC TEMPERATURE FIELD

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Paraboloid mirrors (concentrators) are widely used in solar energy technology, radioastronomy and other branches of science and technology. /26*

The constant improvement of designs, in order to increase the output and efficiency of units, is accompanied by an increase in the requirements placed on the effectiveness of all systems, including paraboloid mirrors. One of the means of increasing the effectiveness of mirrors and reducing the dissipation of output is to increase the precision of manufacture of the reflecting surfaces of the mirrors. In this connection, the problem arises of retaining the initial precision under operating conditions, when the design is subjected to the effect of various external factors. One may take these factors into account by introducing corrections into the control system, which compensate for the changes in the parameters of the mirror, brought about by distortions of the reflecting surface.

The determination of the necessary corrections is possible using measuring systems, but the effectiveness of these systems depends largely on how accurately the behavior of the design will be predicted in various positions and under various environmental conditions.

Units, which operate under the open sky, are subjected to the effects of climatic factors, including solar radiation. In this case, the temperature field of the design, and the field of temperature deformations which corresponds to it, depend on variation in the temperature of the surrounding air, changes in the intensity of solar radiation, the position of the mirror relative to the direc-

*Numbers in the margin indicate pagination in the foreign text.

tion of the solar radiation, and on the wind velocity and direction.

A uniform change in the temperature of the structure causes a similar increase or decrease in its dimensions without distortion of its shape, with the exception of the case when the structure is manfuactured from materials which have different coefficients of thermal expansion.

Non-uniform heating causes temperature deformations, which lead to distortion of the profile of the reflecting surface, a change in the focal length, and deviation of the optical axis of the mirror.

In the given article, we examine axisymmetric temperature deformations of paraboloid mirrors (Fig. 1), brought about by their heating. Such deformations take place, for example, in the case when the optical axis of the mirror is directed towards the sun. /27
In this case, the desired magnitudes are most frequently the deflections and angles of rotation at individual points of the mirror. Also of occasional interest are the magnitudes which characterize the stressed state of the mirror. These include the forces T_1, T_2, Q and the moments M_1 and M_2.

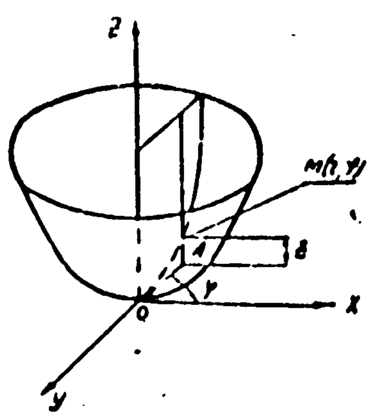

Fig. 1. Paraboloid, closed at the apex: M(r, φ) is a random point on the surface of the mirror; A is the projection of the point M on the plane xy(z=0); φ is the polar angle.

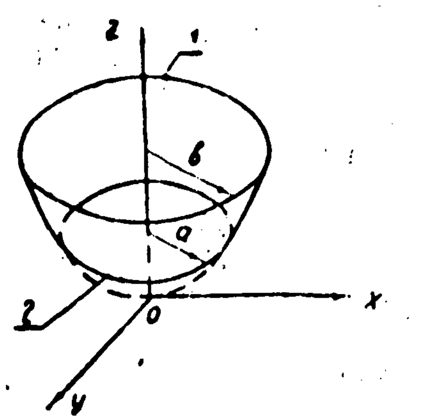

Fig. 2. Paraboloid, open at the apex: 1—upper edge; 2—lower edge.

We will assume that the surface of the paraboloid mirror is an envelope of rotation, the mean surface of which, in cylindrical coordinates, is described by the equation

$$z = a_1 \cdot r + b_1 \cdot r^2 + c_1 \, r^3, \tag{1}$$

where a_1, b_1, c_1 are some constants, which characterize the appearance of the surface of the mirror.

The resolving differential equation of the axisymmetric temperature deformation of the envelope of rotation may be obtained on the basis of Mushtari-Donnel-Vlasov simplifications, with the enlistment of some supplementary assumptions relative to the slope of the envelope [1], [2], [3]. This equation has the form

$$\frac{d^2\Psi}{dr^2} + \frac{1}{r}\frac{d\Psi}{dr} + \left[Eh\gamma^4 i \cdot \frac{1}{r}\frac{dz}{dr} - \frac{1}{r^2}\right]\Psi = -Eh\gamma^4\left(\gamma^4 \frac{dM}{dr} + i\beta_t \frac{dt_{cp}}{dr}\right). \tag{2}$$

Considering relationship (1), we obtain

$$r^2 \frac{d^2\Psi}{dr^2} + r\frac{d\Psi}{dr} + [Eh\gamma^4 i\,(Ar + Br^2 + Cr^3) - 1]\,\Psi = -Eh\gamma^4 r^2\left(\gamma^4 \frac{dM_t}{dr} + i\beta_t \frac{dt_{cp}}{dr}\right), \tag{2'}$$

where $A=a_1$, $B=2b_1$, $C=3c_1$.

The homogeneous differential equation, which corresponds to (2), has the appearance

$$r^2 \frac{d^2\Psi}{dr^2} + r\frac{d\Psi}{dr} + [Eh\gamma^4 i\,(Ar + Br^2 + Cr^3) - 1]\,\Psi = 0. \tag{3}$$ /28

The difficulty of its solution consists of the fact that it may not be reduced to any of the known types of differential equations with variable coefficients.

Insofar as the coefficients of equation (3) are analytical functions of r, one of its partial solutions, according to [4], may be found in the form of an exponential series of the type

$$\Psi_1 = \sum_{s=0}^{\infty} (\alpha_s + i\beta_s)\, r^{s+n}, \qquad (4)$$

where n, α_s and β_s are constants to be determined.

By substituting the magnitude Ψ_1 into (3) and comparing the coefficients with identical exponents of r, we will have n=1. In this case, we will also obtain recurrent relationships for determining the coefficients α_s and β_s. They will take on the form:

$$\begin{cases} \alpha_s = \dfrac{1}{s(s+2)} [a\beta_{s-1} + b\beta_{s-2} + c\beta_{s-3}]; \\ \beta_s = -\dfrac{1}{s(s+2)} [a\alpha_{s-1} + b\alpha_{s-2} + c\alpha_{s-3}], \end{cases} \qquad (5)$$

where $a=Eh\gamma^4A$, $b=Eh\gamma^4B$, $C=Eh\gamma^4C$.

Without disrupting the continuity of the obtained solution, one may assume that $\alpha_0=1$, $\beta_0=0$.

The second partial solution of differential equation (3), according to [4], is found according to the formula

$$\Psi_2 = \Psi_1 \ln r + \sum_{s=0}^{1} (\gamma_s + i\delta_s)\, r^{s-1} + \sum_{s=3}^{\infty} (\gamma_s + i\delta_s)\, r^{s-1}, \qquad (6)$$

where γ_s and δ_s are coefficients which are also to be determined.

The recurrent relationships for determining the coefficients γ_s and δ_s are written in the form

$$\begin{cases} \gamma_s = \dfrac{1}{s(s-2)} [a\delta_{s-1} + b\delta_{s-2} + c\delta_{s-3} - 2(s-1)\alpha_{s-2}], \\ \delta_s = -\dfrac{1}{s(s-2)} [a\gamma_{s-1} + b\gamma_{s-2} + c\gamma_{s-3} + 2(s-1)\beta_{s-2}], \end{cases} \qquad (7)$$

where a, b, c are the very same magnitudes as in the system of equations (5).

With regard for (7), one may represent dependence (6) in the form

$$\Psi_2=\Psi_1 \ln r+\left[\frac{1}{r}\left(1+\frac{b}{a^2} i\right)+\left(-\frac{b}{a}+ia\right)\right]\gamma_0+$$
$$+\sum_{i=1}^{\infty}(\gamma_i+i\delta_i)r^{i-1}, \qquad (8)$$

where $\gamma_0=\dfrac{2a^2}{C^4+b^2}$.

The general solution of the homogeneous differential equation /29 (3) is written in the form

$$\Psi_0=C_{1,0}\,\Psi_1+C_{2,0}\,\Psi_2. \qquad (9')$$

where $C_{1,0}$ and $C_{2,0}$ are complex random constants.

We would note that, with A=C=0, the functions Ψ_1 and Ψ_2 change into $I_1(r\sqrt{T})$ and $Y_1(r\sqrt{T})$, respectively, where I_1 and Y_1 are cylindrical functions of the first order, of the first and second type, of the argument $r\sqrt{T}$.

The partial solution of the heterogeneous differential equation (2') is found according to the general law. Thus, it may be sought according to the method of variation of the random constants, or by representing it in the form of a series according to increasing powers of r.

Thus, the general solution of the heterogeneous differential equation (2) may be represented in the following form:

$$\Psi_1=C_{1,0}\,\Psi_1+C_{2,0}\,\Psi_2+\Psi_r, \qquad (9'')$$

where Ψ_r is some partial solution of the differential equation (2').

Insofar as the random constants $C_{1,0}$ and $C_{2,0}$ and the functions Ψ_1, Ψ_2 and Ψ_r may be represented in the form

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$$C_{1,0} = C_1 + iD_1;$$
$$C_{2,0} = C_2 + iD_2;$$
$$\Psi_1 = \varphi_1 + i\psi_1;$$
$$\Psi_2 = \varphi_2 + i\psi_2;$$
$$\Psi_r = \varphi_r + i\psi_r,$$

the general solution is written in the following manner:

$$\Psi = (C_1\varphi_1 - D_1\psi_1 + C_2\varphi_2 - D_2\psi_2 + \varphi_r) + \\ + i(C_1\psi_1 + D_1\varphi_1 + C_2\psi_2 + D_2\varphi_2 + \psi_r). \qquad (9)$$

The obtained solution contains four random constants; therefore, two each of the boundary value conditions should be fulfilled at each edge of the envelope, which is not closed at the apex (Fig. 2, r=a, r=b).

For the freely-displaced edge, we have

$$M_1 = 0;\ T_1 = 0. \qquad (10)$$

This case of the boundary value conditions corresponds, for example, to a paraboloid mirror attached at the apex. In this case, the conditions (10) correspond to the edge of the mirror which is not attached by any force elements whatsoever, which hinder its free deformation.

For a rigidly fixed edge

$$\nu_1 = 0;\ \varepsilon_2 = 0. \qquad (11)$$

Condition (11) corresponds, for example, to fastening of the mirror at its upper edge. Also possible are combinations of the boundary value conditions (10) and (11).

If the envelope is closed at the apex (Fig. 1), then $C_2 = D_2 = 0$, insofar as $_2(0) = \infty$ and $\psi_2(0) = \infty$. In this case, the general solution contains only two random constants, which are determined from the boundary value conditions on the remaining edge of the envelope, which also solves the posed problem. /30

Thus, dependence (9), together with the boundary value conditions (10) and (11), makes it possible to find the desired function Ψ, via which all of the magnitudes, which are of interest to us for practical calculations, may be expressed.

Thus, the angle of deflection of the random point of the mean surface is determined from the relationship

$$\upsilon = \mathrm{Re}\Psi, \qquad (12)$$

where $\mathrm{Re}\Psi$ is an actual part of the function Ψ.

The deflection is expressed via the angle of deflection according to the formula

$$w = \int \upsilon dr + w_0, \qquad (13)$$

where the constant ω_0 may be placed equal to zero, since it characterizes the overall displacement of the mirror as a rigid whole.

The bending moments and the intersecting force are found from the relationships

$$M_1 = -D\left[\frac{dv}{dr} + \frac{\mu}{r}v\right] + M_t; \qquad (14)$$

$$M_2 = -D\left[\mu\frac{dv}{dr} + \frac{v}{r}\right] + M_t; \qquad (15)$$

$$Q = \frac{dz}{dr}\frac{\Phi}{r} + r\frac{dM_t}{dr}. \qquad (16)$$

The normal forces, which act on the mean surface, are expressed via the function of stresses Φ, which is equal to

$$\Phi = \frac{1}{\gamma^4} Im\,\Psi, \qquad (17)$$

where $\mathrm{Im}\Psi$ is an imaginary part of the function Ψ.

In this case

$$T_1 = \frac{\Phi}{r} - (1+\mu)\beta_t t_{cp}; \qquad (18)$$

$$T_2 = \frac{d\Phi}{dr} - (1+\mu)\beta_t t_{cp}. \qquad (19)$$

The radial displacement of the random point of the mean surface has the form

$$u = \frac{1}{Eh}\left(r\frac{d\Phi}{dr} - \mu\Phi\right) - \frac{dz}{dr}\int v dr - \beta_t t_{cp} r. \qquad (20)$$

The circular deformation and the change in curvature are written in the form

$$\left|\begin{array}{l} \varepsilon_1 = \frac{1}{Eh}(T_2 - \mu T_1) + \beta_t t_{cp}; \\ \varkappa_2 = \frac{1}{D}(M_2 - \mu M_1) + \beta_t \operatorname{grad} t. \end{array}\right. \qquad (21)$$

The examined case of axisymmetric temperature deformation is characteristic for the paraboloid mirrors or units which operate in a "tracking the sun" mode. /31

In this case, deviations of the points of the reflecting surface relative to the like points of the theoretical paraboloid cause a change in the focal length.

This change may be determined as the difference between the focal length of the initial and the approximating surfaces.

The surface of the approximating paraboloid may be calculated on the basis of the obtained temperature displacements of the points of the surface of the given paraboloid, proceeding from the condition of obtaining the least deviations of the points of the surfact from the theoretical profile with a new focal length.

Adopted Designations

r is the radial coordinate of the random point of the mean surface of the mirror; u is the deflection of the random point of the mean surface of the mirror; $\Psi = \upsilon + i\gamma^4\Phi$ is a complex function; υ is the angle of deflection of the mean surface; Φ is the function of stresses on the mean surface of the mirror; $\gamma^4 = 2\sqrt{3(I-\mu^3)}/Eh$ is the coefficient of rigidity; E, μ, β_t are the modulus of elasticity, the Poisson coefficient, and the coefficient of linear expansion of the material; h is the thickness of the envelope; R_1 and R_2 are the main radii of curvature,

$$\frac{1}{R_1} = -\frac{d^2 z}{dr^2}; \quad \frac{1}{R_2} = \frac{1}{r}\frac{dz}{dr};$$

$M_1 = (1+\mu)\, D\beta_1$ gradt=temperature moment: gradt=$\Delta t/h$ is the temperature gradient according to the thickness of the envelope; Δt is the difference in temperatures of the outer and inner surfaces of the mirror; t_{mean} is the temperature of the mean surface; $D = Eh^2/12(1-\mu^3)$ is the cylindrical rigidity; T_1 is the normal radial force; T_2 is the normal circular force; M_1 is the radial bending moment; M_2 is the circular bending moment; Q is the intersecting force; u is the radial displacement; ε_2 is the circular deformation; $\varkappa_2$ is the change in curvature of the envelope in the circular direction.

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